STATISTICAL TECHNIQUES FOR OBJECTIVE CHARACTERIZATION OF MICROWAVE DEVICE STATISTICAL DATA

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ABSTRACT

A comprehensive treatment of statistical metrics for the characterization of microwave device statistical data is presented. The primary aim is to investigate the power of these tests in their ability to faithfully delineate between like and unlike Joint Probability Density Functions (JPDF). This paper shows that adequate techniques are available to solve this problem, and illustrates a novel application of these techniques by distinguishing the statistical difference between two GaAs FET data bases that have identical means, standard deviations, kurtosis, skewness and correlations. Finally, we verify our characterization approach by design centering a small-signal amplifier, both with and without the use of statistically characterized device data.

I. INTRODUCTION

Over the past two decades, a great deal of work in the area of statistical design has been accomplished [1-4]. However, only a small fraction of this literature addresses the accuracy of the statistical description used in the yield optimizer [5-10]. Unfortunately, these works offer only crude (possibly inaccurate), uncharacterized models of the underlying statistics. Without objective and accurate characterization of input parameter statistics, the outcome of the design is subject to greater uncertainty, even with robust statistical optimization algorithms. Spanos and Director were the first (with respect to circuit design) to point out that a series of independent univariate statistical tests will likely yield poor simulation results [10]. They called for a *multivariate* test, i.e., a test that will accept or reject the null hypothesis of moment equivalence for *all* the variables at once. Realizing this, they worked around the problem by using a simplified (locally linear) model.

In essence, the univariate tests (Kolmogorov-Smirnov, mean, std. dev. kurtosis, skewness, Chi-squared, correlations, etc.) are necessary but not sufficient indicators of moment equivalence for JPDF's. Meehan and Collins provided evidence of this by showing that two data sets of FET equivalent circuit parameters, one measured and the other synthesized and having identical distributions and correlations, produced sets of S-parameters with *different* univariate test results [11].

The following section provides insight into the available multivariate test statistics. Section III highlights an example using one of the new multivariate tests with actual Foundry GaAs FET data. Section IV presents a statistical design example using both characterized and uncharacterized device data. Finally, section V summarizes our findings.

II. STATISTICAL TESTS FOR DEVICE STATISTICAL DATA

For one dimensional data, there are several tests which can distinguish two samples, such as the Chi-squared and Kolmogorov-Smirnov (K-S) tests. However, these tests either do not scale directly to higher dimensions, or else they do not have adequate "power" in higher dimensions. Two promising solutions to this problem are those presented in Friedman [12] (generalized K-S test) which use minimal spanning trees to generalize one-dimensional tests to higher dimensions, and a new approach based on nearest neighbor type coincidences presented in Schilling [13] and Henze [14].

2.1 Generalized Kolmogorov-Smirnov Test. The Kolmogorov-Smirnov test compares two sets of samples by measuring the maximum deviation between the cumulative distributions of the samples. In one dimension, the K-S test works by ordering the combined samples, and measuring the percentage of samples of the opposite type less than each sample. The significant statistic is the maximum difference between the percentage of samples from the opposite set which are lower in order than a given sample.

In order to make the K-S test work in higher dimensions, it is necessary to define an ordering on samples which is meaningful in higher dimensions. In Friedman this is done by using minimal spanning trees traversed in a "height directed preorder" pattern [12]. A Spanning Tree is a non-cyclic graph containing all points in the space. A Minimal Spanning Tree is a Spanning Tree where the edges of the tree are weighted by the distance between points, and the sum of the weights is a minimum. The traversal of a MST is a recursive algorithm as follows: visit the root of the tree; then traverse the subtrees of the root in order of least to greatest maximum depth of the subtree. The traversal of the Minimal Spanning Tree defines an order which can then be used by the one-dimensional K-S test. Algorithms for constructing Minimal Spanning Trees can be found in most common algorithm books.

2.2 Nearest Neighbor Test. A new approach to solving the two-sample problem for higher dimensions is presented in Schilling [13] and Henze [14], and is based on the number of nearest neighbor type coincidences. The basic idea is to find the k nearest neighbors in the space according to a given distance measure. A statistic is then computed by the following formula [13]:

$$T_{k,n} = \frac{1}{nk}\sum_{i=1}^{n}\sum_{r=1}^{k} I_i(r), \tag{1}$$

where n is the number of samples, k is the number of nearest neighbors, and $I_i(r)$ is unity if the r^{th} nearest neighbor is of the same type and zero otherwise. In Schilling, the asymptotic distribution of the statistic is found to be gaussian for a Euclidean distance measure [13]. In Henze, the asymptotic distribution of the statistic is given for any distance measure [14]. Henze's asymptotic distribution is somewhat more complicated than Schilling's, and will not be discussed here.

III. MULTIVARIATE CHARACTERIZATION OF FET DATA

In this example of statistical characterization, we use the measured GaAs FET data from [15]. Using a careful synthesis process, we produce another FET equivalent circuit parameter data base having closely matched univariate statistics. It is important to note that the synthesized model makes no simplifying assumptions about the distributions or correlations - these are very similar to those measured. Table 1 shows the results from the univariate K-S test which indicate that the distributions for the measured and synthesized data are identical.

Ri	-> Var 0 - KS statistic = 0.015909 with confidence = 1.000000
Rds	-> Var 1 - KS statistic = 0.011818 with confidence = 1.000000
gm	-> Var 2 - KS statistic = 0.011818 with confidence = 1.000000
Cds	-> Var 3 - KS statistic = 0.010909 with confidence = 1.000000
Cgs	-> Var 4 - KS statistic = 0.012273 with confidence = 1.000000
Cdg	-> Var 5 - KS statistic = 0.013182 with confidence = 1.000000
t	-> Var 6 - KS statistic = 0.013636 with confidence = 1.000000
Gdg	-> Var 7 - KS statistic = 0.014091 with confidence = 1.000000

Table 1 Univariate K-S test on measured vs. synthesized FET parameter data sets.

The correlation matrix for 88 measured devices:

	0	1	2	3	4	5	6	7
0	1.000							
1	0.487	1.000						
2	0.068	0.597	1.000					
3	-0.851	-0.454	-0.279	1.000				
4	0.259	0.601	0.820	-0.485	1.000			
5	-0.067	-0.692	-0.165	-0.051	-0.144	1.000		
6	-0.062	0.233	0.080	0.048	0.446	-0.288	1.000	
7	0.677	0.527	0.130	-0.629	0.082	-0.350	-0.403	1.0

The correlation matrix for 200 synthesized devices:

	0	1	2	3	4	5	6	7
0	1.000							
1	0.421	1.000						
2	0.002	0.556	1.000					
3	-0.809	-0.361	-0.185	1.000				
4	0.266	0.555	0.750	-0.450	1.000			
5	-0.065	-0.690	-0.178	-0.070	-0.067	1.000		
6	-0.104	0.204	0.020	0.145	0.363	-0.274	1.000	
7	0.637	0.481	0.118	-0.621	0.086	-0.320	-0.373	1.0

Table 2 Pairwise Correlations on measured and synthesized FET parameter data sets.

Table 2 shows the pairwise correlation matrix for measured and synthesized data, again with similar results. When the multivariate test statistic (1) is used, we obtain a confidence level (consult [13] for computational details on μ_i and σ_i):

$$\left| erf\left(\frac{(nk)^{1/2}(T_{kn}-\mu_k)/\sigma_k}{\sqrt{2}}\right)\Bigg|_{k=8}\right| = 0.99983. \qquad (2)$$

This indicates that we reject the hypothesis that the measured and synthesized data are the same, with a 99.98% confidence level. This helps to explain the results given in [11, 15], where univariate characteristics of the measured S-parameter data did not compare well with the synthesized S-parameter characteristics, generated from the FET parameter data as given above. If we examine scatter plots of the FET data, we can appreciate why the multivariate test rejects Ho. Figure 1 shows Ri vs. tau for a) the measured data set, and b) the synthesized data set. It is easy to recognize areas where combinatorial discrepancies are prevalent, yet there is excellent agreement between the distributions and correlations involved. This is the main theme: ensembles of univariate test statistics are not suitable for the characterization of multivariate Joint Probability Density Functions.

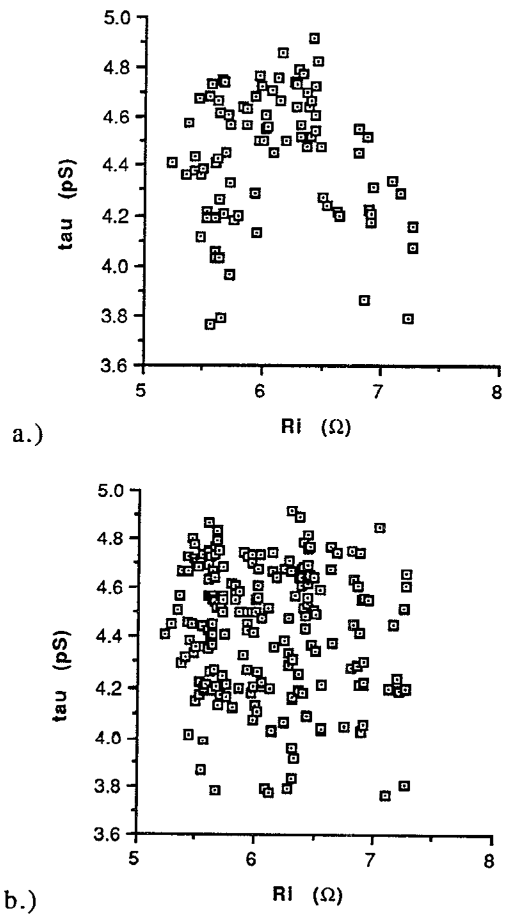


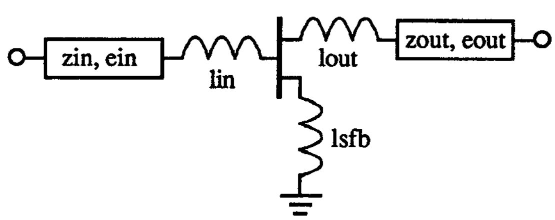
Figure 1 Scatter Plot of tau vs. Ri for a) measured, and b) synthesized FET parameter data sets.

Finally, we exercised the multivariate test on the synthesized FET data set "split in half." Thus, we would expect this test would indicate that the data sets are from the same population. This test indicated statistical equivalence with an 88% level of confidence.

Next we illustrate the implications of device statistical characterization by design centering a single FET amplifier.

IV. DESIGN CENTERING WITH CHARACTERIZED DEVICE DATA

The following example is taken from Purviance, *et al.* [6]. The circuit and optimization specifications for this example are given in Figure 2. Starting from the nominally optimized component values as given in [6], design centering using Touchstone™ [16] was performed twice using the following assumptions on the FET model statistics: 1) use multivariate characterized (measured) device data, and 2) use univariate characterized device data. In other words, we use the FET data sample sets from the previous section. The designable parameters are zin, zout, ein, eout, lin, lout, lsfb, and are modeled as independent uniform variables with 10% tolerance limits. Tables 3 and 4 summarize the results of this centering exercise. Table 3 shows yield estimates derived from 5000 Monte Carlo trials. The yield estimates given use the multivariate characterized (measured) data. Note that yield is improved using both characterized and uncharacterized FET data. However, we should point out that the designer using multivariate characterized device data leaves much less to chance. Another important result here is that both the yield estimate and the design center are affected by the choice of device statistics.



Optimization SPECS			
	S11	S22	S21
Nominal	<-10dB	<-10dB	=15dB
Yield	<-8dB	<-8dB	<16, >14dB

Figure 2 Single FET 3.8-4.2 Ghz. Amplifier used in the Design Centering Example.

YIELD (5000 trials - Monte Carlo)		
	Before Centering	After Centering
Univariate characterization	31.4%	50.1%
Multivariate characterization	31.4%	58.5%

Table 3 Yield estimates before and after design centering.

Parameter	Univariate characterization	Multivariate characterization	Delta %
zin	35.52 Ω	38.15 Ω	7.87
ein	83.78 deg	84.97 deg	1.42
lin	3.21 nH	3.23 nH	2.43
lsfb	0.55 nH	0.57 nH	4.56
lout	7.88 nH	8.48 nH	7.56
zout	82.07 Ω	86.05 Ω	4.85
eout	93.60 deg	99.80 deg	6.62

Table 4 Design centering results for the single FET amplifier.

V. CONCLUSIONS

We have presented techniques for objectively characterizing multivariate statistical device data. These tests are straightforward to implement, and yield high power to distinguish Joint Probability Density Functions with calculable certainty. We have illustrated the application of these techniques by distinguishing the statistical difference between two GaAs FET data bases that have identical means, standard deviations, kurtosis, skewness and correlations. Finally, we verified our approach by design centering a small-signal amplifier, both with and without the use of statistically characterized device data. The centering example shows that not only yield estimates are affected by the accuracy of device statistics, but also the design center.

These techniques should find extensive use not only in statistical device characterization problems, but also in:

a) statistical process characterization [10],

b) statistical design algorithms [16], and

c) statistical interpolation algorithms [17].

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